-une 1993 Journal Article

THE METHOD OF RATIOS

2

PR: 9993
TA: LA
WU: BS

David Negron, Jr., Salvatore Alfano, Daniel Donald Wright III

United States Air Force Academy
Colorado Springs, Colorado 80840

Phillips Laboratory
3550 Aberdeen Avenue, SE
Kirtland AFB, NM 87117-5776

PL-TR--93-1019

Published in Journal of the Astronautical Sciences, Vol. 40, No. 2, Apr-Jun 92, pp. 297-309.
Lt Col Alfano is presently assigned to PL/VTA, Kirtland AFB, NM 87117-5776.

This paper presents a detailed algorithm to compute a mean visibility report between a range-restricted ground station and a space object, generated by the method of ratios. This method is based upon the limiting orbital characteristics of a space object versus a ground station, which are used to generate a visibility report from an analytical model. For this study, the visibility report is defined as the mean minutes-per-day the space object is above the site's horizon and within sensor range. The computer simulation models orbital motion using first order secular perturbations caused by mass anomalies. The method of ratios reduces computation time by over 97% when the results are compared to a visibility *truth* table created using the satellite rise-set algorithm developed by Alfano et al. This method can also be used to evaluate all objects in a space catalogue against a network of range-restricted ground stations. Operations personnel can then use this master visibility report to maintain a current database of all space objects visible to a particular ground station, which is central for space surveillance radar sensors to perform efficient tracking.

mathematical models, numerical methods and procedures, models, visual surveillance, tracking, visibility, air-to-space, Earth (planet) observation

14

Unclassified Unclassified Unclassified SAR

The Journal of the Astronautical Sciences, Vol. 40, No. 2, April-June 1992, pp. 297-309

The Method of Ratios

David Negron, Jr.,[1] Salvatore Alfano,[2] and Daniel Donald Wright III[3]

Abstract

This paper presents a detailed algorithm to compute a mean visibility report between a range-restricted ground station and a space object, generated by the method of ratios. This method is based upon the limiting orbital characteristics of a space object versus a ground station, which are used to generate a visibility report from an analytical model. For this study, the visibility report is defined as the mean minutes-per-day the space object is above the site's horizon and within sensor range. The computer simulation models orbital motion using first order secular perturbations caused by mass anomalies. The method of ratios reduces computation time by over 97% when the results are compared to a visibility *truth* table created using the satellite rise-set algorithm developed by Alfano et al. This method can also be used to evaluate all objects in a space catalogue against a network of range-restricted ground stations. Operations personnel can then use this master visibility report to maintain a current database of all space objects visible to a particular ground station, which is central for space surveillance radar sensors to perform efficient tracking.

Introduction

Space operations personnel performing space track have enjoyed the luxury of working with a small database of only a few thousand space objects. However, international interest in space and the increased domestic reliance on orbital remote sensing and communications has caused the population of artificial Earth satellites, which is continuously growing, to total more than seven thousand objects. This growth of orbiting objects requires a revised strategy in tasking range-restricted radar sensors and maintaining their orbital element database. This becomes tremendously important for sensor sites with the dual responsibility of space track and missile warning. Each time a site with these responsibilities identifies a space object with elements that do not match a member of its data-

[1]Assistant Professor, Astrodynamics Division, Department of Astronautics. Address: USAFA/DFAS, USAFA, CO 80840.
[2]Associate Professor, Astrodynamics Division Chief, Department of Astronautics, Member AAS. Address: USAFA/DFAS, USAFA, CO 80840.
[3]Space Operations Officer, 5th Satellite Control Squadron. Address: 5th SCS, Fairchild AFB, WA 99011-5000.

base, operations personnel must transmit a message to alert Cheyenne Mountain AFB orbital analysts of the uncorrelated contact. If the site's database is not current this becomes a frequent task.

Sensor tasking and database maintenance require the processing of each object in a space catalogue against a network of sites. The processing algorithm must be both efficient and robust because a catalogue can include thousands of objects spanning the range of closed orbits, as shown in Figs. 1 through 3. One possible method to create a visibility report is to use a step-by-step propagation scheme to simulate a sufficiently large number of passes of a space object versus a station and record statistics about the angular separation of their Earth centered inertial position vectors. A similar method involves creating a report of topocentric slant range and azimuth-elevation angles to determine if and when an object becomes visible to a site. Two drawbacks common to these methods are lengthy computation time and the dependence on periapsis rate to arrive at a reasonable answer. As periapsis rate tends to zero, the simulation runtime becomes prohibitively long because the accuracy of the visibility report depends on modeling all possible passes of an object versus a station. A geometric solution to determine if there is visibility between a range-restricted station and a space object involves creating a right triangle at the site with hypotenuse equal to the object's maximum radial distance from the geocenter. If the vertical member of the triangle becomes greater than one Earth radii, the space object will have a positive topocentric elevation. This method gives no insight about contact duration, especially for a range-restricted site.

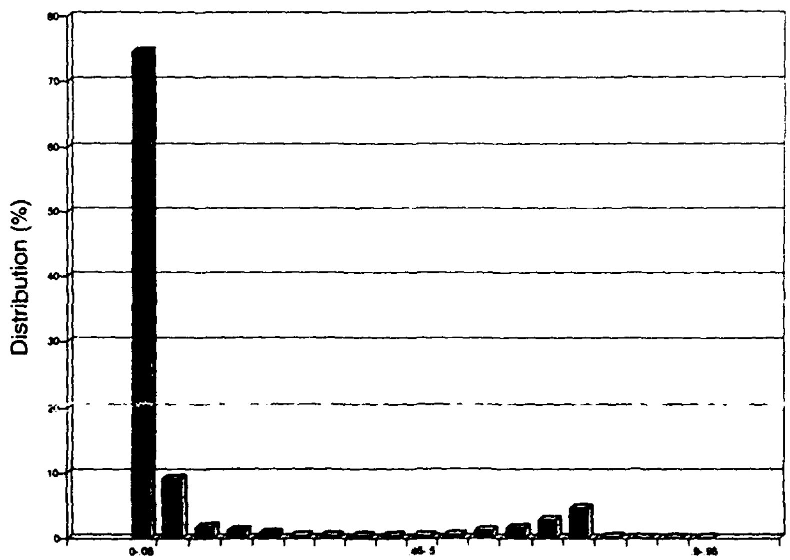

FIG. 1. Space Object Distribution versus Orbital Eccentricity.

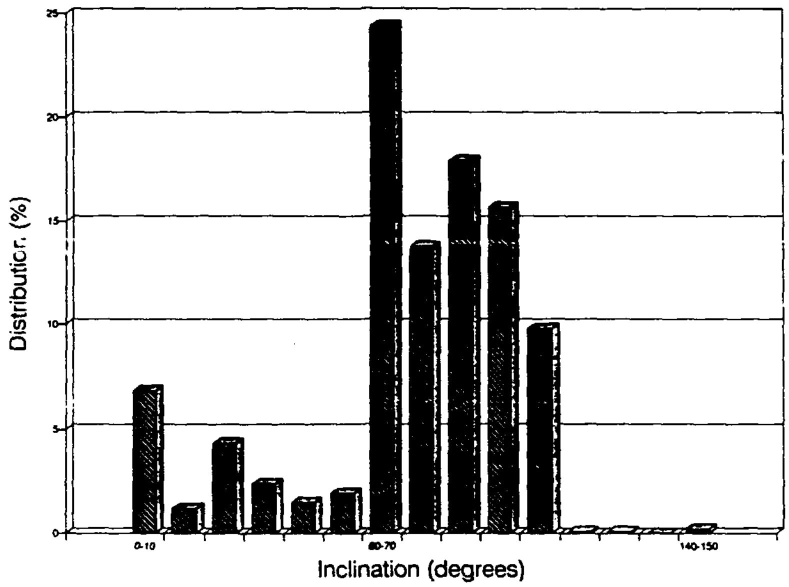

FIG. 2. Space Object Distribution versus Orbital Inclination.

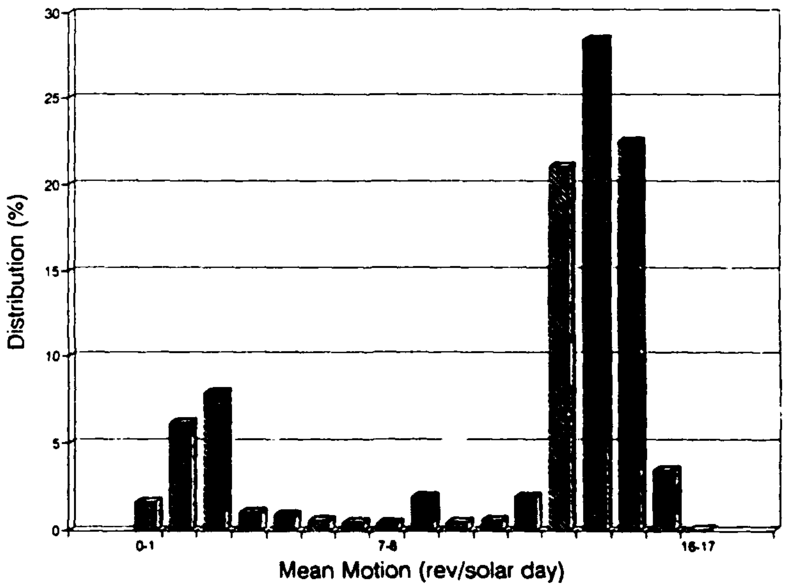

FIG. 3. Space Object Distribution versus Orbital Mean Motion.

The method used to determine sensor tasking and database maintenance must minimize simulation runtime while maximizing the accuracy of the site-object visibility report. The number of artificial Earth satellites, which is continuously growing, renders the multiple pass and topocentric range-angle simulation an expensive choice. The published literature about space sensor and land mass line-of-sight has centered about satellite deployment strategies to optimize remote sensing parameters for local and global Earth coverage. Examples of this work include [1–4] which address how to create optimal geometry for visibility between a spaceborne sensor and an Earth target. This paper addresses the opposite problem because the sensing platform is now Earth located and its sensor range is limited. The objective of this paper is to present an analytical model that accomplishes sensor tasking and database maintenance efficiently enough to execute on a personal computer.

The Ratio Equation

Presented below is a geometric model to formulate the ratio equation. Hayes [1, 2] used the concept of ratios to analytically study the contact between a spaceborne sensor and an Earth target. What follows is an independent derivation of this equation and a detailed description of its use to detemine the visibility between a range-restricted site and a space object. The fundamental unit of length for the ensuing equations is one Earth radii.

The method of ratios is based on the assumption that the space object is visible to all sites with latitude L and sensor range ρ_{MAX} with similar frequency. Considering the Earth as a fixed sphere, the space object is advanced in its orbital plane with visibility from the range-restricted site being evaluated at n discrete points. The mean anomaly is chosen as the controlling variable for this algorithm because a constant step size allows the object to linger over a hemisphere as determined by its orbital eccentricity and periapsis. To account for apsidal drift, the argument of periapsis is initialized in the southern hemisphere and the visibility contact determined for one orbit. This process is repeated, moving periapsis in m discrete increments until completing one cycle. The mean anomaly and periapsis step sizes are designer chosen; for this study n is 100 and m is 8. Advancing periapsis by 45° results in symmetry between the range-restricted site and the space object, with only five iterations needed to complete the simulation.

As shown in Fig. 4, the $(\zeta_1, \zeta_2, \zeta_3)$ coordinate system is used to express the geometry between a site with sensor range ρ_{MAX} and a space object. This coordinate system is created such that its principal axis is aligned on the sensor meridian, the equator serves as the fundamental plane, and the origin is at the geocenter. Thus, from Fig. 4, the sensor site and space object position vectors are given by

$$\mathbf{R}_{SITE} = \cos(L)\hat{\boldsymbol{\zeta}}_1 + \sin(L)\hat{\boldsymbol{\zeta}}_3, \tag{1}$$

and

$$\mathbf{R} = R\cos(L')\cos\left(\frac{\theta}{2}\right)\hat{\boldsymbol{\zeta}}_1 + R\cos(L')\sin\left(\frac{\theta}{2}\right)\hat{\boldsymbol{\zeta}}_2 + R\sin(L')\hat{\boldsymbol{\zeta}}_3, \tag{2}$$

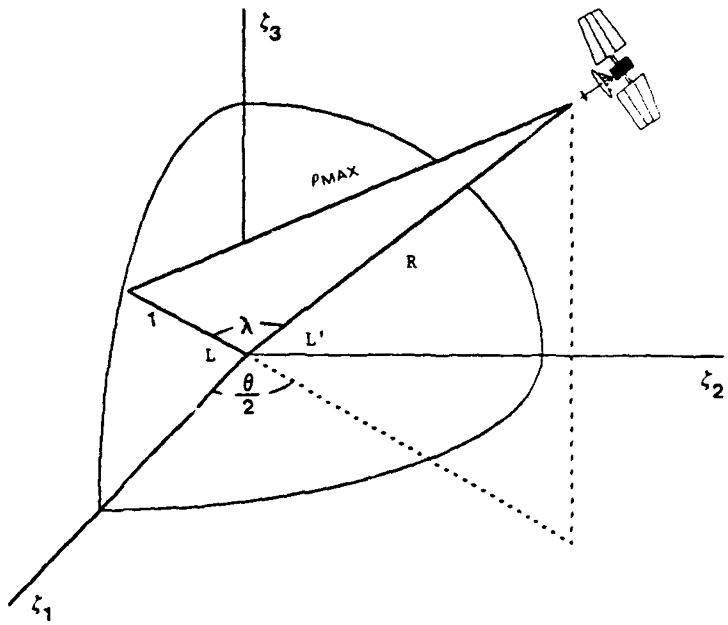

FIG. 4. Geometry Between a Range-Restricted Sensor Site and a Space Object Expressed in the $(\zeta_1, \zeta_2, \zeta_3)$ Coordinate System.

where L and L' are the latitudes of the site and object, respectively, and $\theta/2$ is the angular measure between $\mathbf{R}$ and $\mathbf{R}_{SITE}$ in the fundamental plane. From the inner product, the angle λ is given by

$$\cos(\lambda) = \frac{\mathbf{R}_{SITE} \cdot \mathbf{R}}{R} = \cos(L)\cos(L')\cos\left(\frac{\theta}{2}\right) + \sin(L)\sin(L'). \tag{3}$$

Also from the law of cosines

$$\cos(\lambda) = \frac{R^2 + 1 - \rho_{MAX}^2}{2R}. \tag{4}$$

Equating these two expressions for $\cos(\lambda)$ and solving for θ yields the ratio equation

$$\theta = 2\cos^{-1}\left(\frac{(R^2 + 1 - \rho_{MAX}^2)/(2R) - \sin(L)\sin(L')}{\cos(L)\cos(L')}\right). \tag{5}$$

The above equation is valid when the object is above the site's horizon and within sensor range, as defined by the constraints

$$R\cos(L - L') \geq 1.0, \tag{6}$$

and

$$R^2 + 1 - 2R\cos(L - L') \leq \rho_{MAX}^2. \tag{7}$$

For a site with a sufficiently large sensor range viewing a low-Earth orbiting space object, it is possible for the object to go below the horizon prior to exceeding the site's maximum sensor range. Should this occur, the constraint

$$\rho_{MAX}^2 > (R^2 - 1) \tag{8}$$

is satisfied and the ratio equation becomes

$$\theta = 2 \cos^{-1}\left(\frac{1/R - \sin(L)\sin(L')}{\cos(L)\cos(L')}\right). \tag{9}$$

Since the object is assumed to be visible with similar frequency, it's likelihood of being seen by a specified site is given by

$$P(\text{visibility}) = \frac{\theta}{2\pi}. \tag{10}$$

The ratio equation presents numerical difficulties when the absolute value of the inverse cosine argument becomes greater than one, which can occur frequently for a site with infinite sensor range placed near a pole. To avoid this condition, it is necessary to understand how the argument represents the physical relationship between the range-restricted site and the space object. If the inverse cosine argument becomes greater than or equal to one, then the object is completely out of view and θ is zero. If the argument becomes less than or equal to negative one, then any site at that latitude can view the object and θ becomes 2π.

Visibility Filter

An algorithm to filter space objects that are never visible to a sensor site with latitude L and range ρ_{MAX} is needed to eliminate unnecessary computer processing. For a visibility report to be nonzero, the space object must (1) have a positive topocentric elevation at some point in its orbit, and (2) be within sensor range. Equations for prescreening each of these constraints are presented below.

To determine if an object has a positive topocentric elevation, compute the minimum geocentric range,

$$R_{MIN} = \sec(\alpha), \tag{11}$$

where

$$\alpha = \begin{cases} |L| - L'_{MAX} & |L| > L'_{MAX} \\ 0 & \text{otherwise} \end{cases}, \tag{12}$$

and

$$L'_{MAX} = \begin{cases} \text{inclination} & \text{direct orbit} \\ \pi - \text{inclination} & \text{indirect orbit} \end{cases}. \tag{13}$$

Also, the maximum geocentric range, R_{MAX}, for an orbit with L'_{MAX} is given by

$$R_{MAX} = \begin{cases} \cos(\alpha) + \sqrt{\rho_{MAX}^2 - \sin^2(\alpha)} & \rho_{MAX}^2 \geq \sin^2(\alpha) \\ 0 & \text{otherwise} \end{cases}. \tag{14}$$

Thus, if

$$a(1 + e) < R_{MIN} \tag{15}$$

or

$$a(1 - e) > R_{MAX}, \tag{16}$$

where a is the orbital semi-major axis and e is the eccentricity, then the object is never visible to the site, resulting in a visibility report of zero contact time.

Visibility Criterion

As stated previously, the method of ratios is based on the assumption that the space object is visible to all sites with latitude L and sensor range ρ_{MAX} with similar frequency. To determine if this assumption is met, a visibility criterion is developed below.

An object is visible to all sites on a specific latitude with similar frequency if it is seen by the ith and jth sites before the groundtrace pattern repeats. Define θ_{MAX} as the maximum value of all discrete values of θ which are available from computing the ratio equation. If θ_{MAX} is 2π, then at some point in its orbit the object is visible to all sites with latitude L and sensor range ρ_{MAX}; hence the visibility criterion is satisfied. For θ_{MAX} less than 2π the minimum number of orbital revolutions for complete latitudinal viewing becomes

$$N_{\theta_{MAX}} = \frac{2\pi}{\theta_{MAX}}. \tag{17}$$

To determine if the groundtrace pattern repeats before obtaining complete latitudinal viewing, it is necessary to compute the absolute movement of the ascending node per orbital revolution, $\Omega_{\oplus}$, which is given by

$$\Omega_{\oplus} = |\omega_{\oplus} - \dot{\Omega}| I\!P, \tag{18}$$

where $\omega_{\oplus}$ is the rotation rate of the Earth, $\dot{\Omega}$ is the ascending node rate in inertial space caused by the Earth's oblateness, and $I\!P$ is the orbital period. Rounding $\Omega_{\oplus}/\theta_{MAX}$ to the nearest integer gives the minimum number of revolutions of the ascending node about a rotating earth, $N_{\Omega_{\oplus}}$, to ensure complete latitudinal viewing. Numerically this is determined from the equation

$$x_j = \frac{2\pi j}{\Omega_{\oplus}}, \qquad j = 1, 2, \ldots, N_{\Omega_{\oplus}}, \tag{19}$$

where if any x_j is an integer in the range 0 to $N_{\theta_{MAX}}$, exclusive, then the orbital trace repeats too soon. An algorithm for the visibility criterion is given in Appendix B.

Simulation Results

For this study, the truth data is generated by the method of parabolic blending developed by Alfano et al. [5], to determine the rise-set times of satellite-ground station visibility periods. The time step used for the blending is 250 seconds with the object and site initialized at the vernal equinox. The orbital simulation to generate the truth data includes first order secular perturbations caused by mass anomalies [6] where the rate terms are:

$$\bar{n} = n_0\left[1 + \frac{3}{2}J_2\frac{\sqrt{1-e^2}}{p^2}\left(1 - \frac{3}{2}\sin^2 i\right)\right] \tag{20}$$

$$\dot{\Omega} = -\left(\frac{3}{2}\frac{J_2}{p^2}\cos i\right)\bar{n} \tag{21}$$

$$\dot{\omega} = \left(\frac{3}{2}\frac{J_2}{p^2}\left[2 - \frac{5}{2}\sin^2 i\right]\right)\bar{n}, \tag{22}$$

where $\bar{n}$ is the anomalistic mean motion, n_0 is the mean motion at epoch, J_2 is the second harmonic coefficient, p is the semi-latus rectum, e is the eccentricity, i is the inclination, $\dot{\Omega}$ is the nodal rate, and $\dot{\omega}$ is the periapsis rate.

Classical orbital elements from the United States Space Command space object catalogue are used as test data for this study, and are summarized in Table 1. This subset of classical elements is derived by sorting the catalogue, which includes seven thousand objects, by eccentricity, inclination, and mean motion. For completeness the first, central, and last orbital element set from each sort result is then processed against the sites listed in Tables 2 through 5. The sensor site latitudes and range restrictions for the test data are designer chosen to demonstrate the effectiveness and versatility of the method of ratios; hence, these sites are fictitious. This method reduced computation time by 97.65% when compared to parabolic blending for a three month interval; a time savings of 99.41% was realized if the truth data was computed for an entire year.

To assist operations personnel with interpreting the method of ratios results, a column entitled "Confidence" is included with each visibility report. Each entry in the confidence column is made up of a set of flags indicating the results of three tests: (1) visibility, (2) critical inclination, and (3) non-geostationary orbit. These tests serve to alert the operator of repeating, frozen, and geostationary orbits, respectively. Because satellites are deployed with only finite precision, the criteria to declare an orbit inertially or Earth-fixed are designer chosen. For this

TABLE 1. Classical Orbital Elements ($\omega = \Omega = MA = 0°$)

Object	N(rev/solar day)	e	i(deg)
1	1.00272141	.0000032	0.0956
2	8.36589235	.0080158	90.0175
3	0.24891961	.9363060	64.9874
4	0.21467209	.0668128	57.3500
5	13.37659679	.0145072	90.2619
6	16.09769232	.0078742	82.8709
7	1.00271920	.0003109	0.0099
8	12.41552416	.0036498	74.0186
9	13.84150848	.0048964	144.6414

TABLE 2. Visibility Report (min/day) for Site at 80° Latitude with 1,000 KM Range

Space Object	0.5 years	1.0 year	5 years	Ratio method	Confidence
1	0.0	0.0	0.0	0.0	—
2	0.0	0.0	0.0	0.0	—
3	0.0	0.0	0.0	0.0	—
4	0.0	0.0	0.0	0.0	—
5	0.0	0.0	0.0	0.0	—
6	32.1	32.0	32.2	32.9	YYY
7	0.0	0.0	0.0	0.0	—
8	0.0	0.0	0.0	0.0	—
9	0.0	0.0	0.0	0.0	—

TABLE 3. Visibility Report (min/day) for Site at 40° Latitude with 2,000 KM Range

Space Object	0.5 years	1.0 year	5 years	Ratio Method	Confidence
1	0.0	0.0	0.0	0.0	—
2	0.0	0.0	0.0	0.0	—
3	0.0	0.0	0.0	0.0	—
4	0.0	0.0	0.0	0.0	—
5	17.4	17.4	17.4	17.4	YYY
6	23.2	23.1	23.2	23.0	YYY
7	0.0	0.0	0.0	0.0	—
8	11.1	11.1	11.1	11.3	YYY
9	27.5	27.5	27.5	27.3	YYY

TABLE 4. Visibility Report (min/day) for Site at 0° Latitude with 4,000 KM Range

Space Object	0.5 years	1.0 year	5 years	Ratio Method	Confidence
1	0.0	0.0	0.0	0.0	—
2	3.3	3.3	3.4	3.5	YYY
3	0.3	0.3	0.3	0.3	YNY
4	0.0	0.0	0.0	0.0	—
5	69.8	69.8	69.8	69.5	YYY
6	17.4	17.3	17.2	17.5	YYY
7	0.0	0.0	0.0	0.0	—
8	68.0	68.0	68.0	67.8	YYY
9	117.3	117.4	117.4	116.6	YYY

study, an orbit with an inclination within two degrees of the critical inclination and an eccentricity greater than 0.1 fails the second test, an example being space object #3. Also, an orbit fails the non-geostationary test if it has an inclination less than 10 degrees, an eccentricity less than 0.001, and a period within 15 seconds of the sidereal period. Processing the Space Command catalogue

TABLE 5. Visibility Report (min/day) for Site at −60° Latitude with 1,000,000 KM Range

Space Object	0.5 years	1.0 year	5 years	Ratio Method	Confidence
1	1440.0	1440.0	722.9	579.1	NYN
2	379.5	384.3	384.2	384.5	YYY
3	656.6	662.2	643.7	687.5	YNY
4	685.9	685.4	687.5	687.1	YYY
5	178.9	182.9	183.2	183.0	YYY
6	37.6	37.7	37.7	37.4	YYY
7	1440.0	1440.0	720.8	579.1	NYN
8	188.8	188.8	189.5	189.6	YYY
9	25.1	24.9	24.9	25.0	YYY

against these two tests indicate that five percent of the catalogued objects fail the critical inclination test and two percent fail the non-geostationary test. It is worthy to note that a circular or inertially fixed orbit requires the processing of only one orbital revolution to produce an accurate visibility report. The reports generated by the method of ratios and the multiple pass simulation are nearly identical for a space object that passes all three tests, indicated by YYY in Tables 2 through 5. Should the visibility prescreen determine zero contact, a field of dashes appears in the confidence column.

Closing Remarks

This paper presents an analytical model to create a visibility report between a range-restricted ground station and a space object based upon the method of ratios. The method easily lends itself to implementation on a personal computer, processing thousands of objects spanning the range of closed orbits against any site latitude. The method of ratios can also be used to perform parametric studies; identifying the payoff of various upgrade strategies for a tracking station network. Figure 5 shows an example of this application by varying site latitude and sensor range using mean minutes per day as the performance index. Whether attempting to quantify the cost per unit increase in ground system performance or allocating funds to improve or maintain a network of stations, the merits of this tool are easily recognized.

Also, the method of ratios can be used to identify the optimal site latitude to track a class of orbits. Figure 6 illustrates this process by holding sensor range fixed while varying the site latitude. Depending on the satellite viewing priority, this tool can help mission designers position a ground station to maximize the mean cumulative visibility or reposition mobile tracking terminals to maximize contact once a space vehicle has maneuvered. This method can also be used in selecting orbital characteristics to maximize contact with a range-restricted site, supporting space-mission designs for highly maneuverable vehicles.

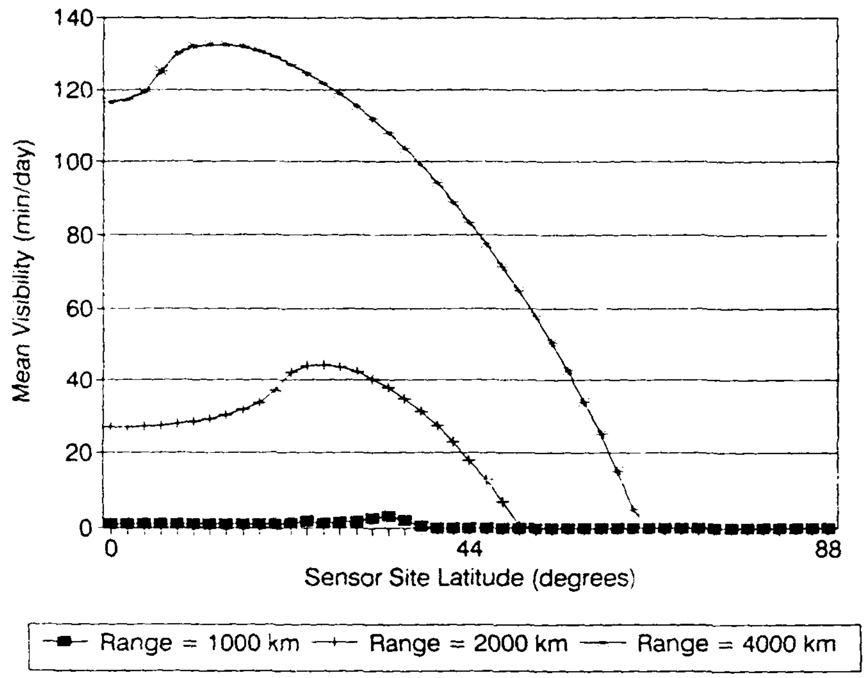

FIG. 5. Visibility Report for Space Object #9 versus Varying Site Latitude and Sensor Range.

Appendix A

Algorithm for the Method of Ratios:

1. Set L equal to the site latitude.
2. Set L' equal to the object sublatitude.
3. Set R equal to object geocentric range.
4. If $R\cos(L - L') < 1.0$, then stop; object is below site horizon, $\theta = 0$.
5. If $\{R^2 + 1 - 2R\cos(L - L')\} > \rho_{MAX}^2$, then stop; object is beyond site range, $\theta = 0$.
6. Compute the numerator of the arc cosine argument. If $\rho_{MAX}^2 > (R^2 - 1)$, then

$$NUM = 1/R - \sin(L)\sin(L')$$

else

$$NUM = \frac{R^2 + 1 - \rho_{MAX}^2}{2R} - \sin(L)\sin(L').$$

7. Compute the denominator, $DEN = \cos(L)\cos(L')$.
8. If DEN is less than some small tolerance or if $|NUM/DEN| \geq 1$, then

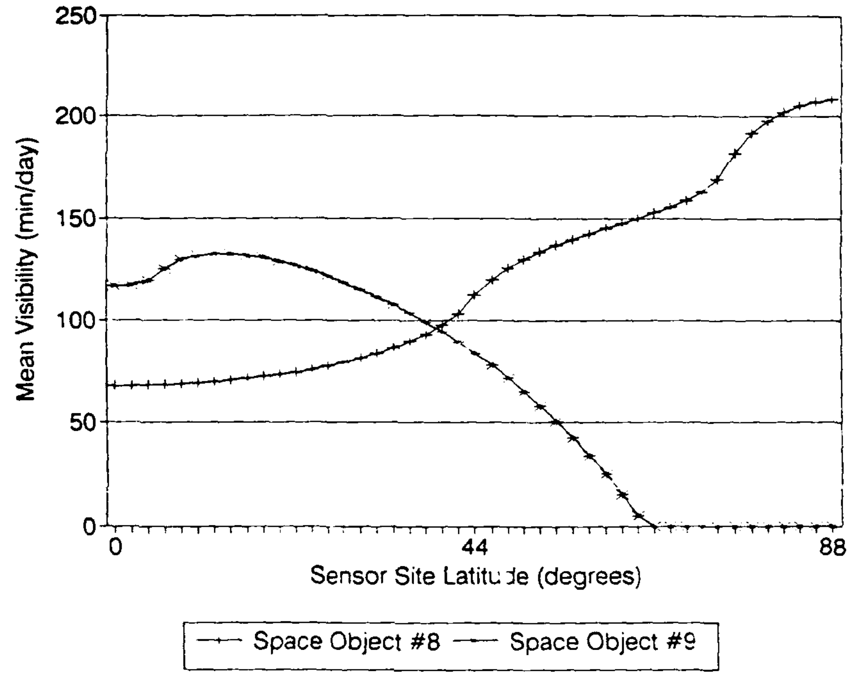

FIG. 6. Visibility Report for Space Objects #8 and #9 versus Varying Site Latitude and 4000 km Sensor Range.

$$\theta = \pi - \pi\ \mathrm{SIGN}(NUM)$$

else

$$\theta = 2\cos^{-1}\left(\frac{NUM}{DEN}\right).$$

9. The likelihood of the object being visible to the range-restricted site becomes

$$P(\text{visibility}) = \frac{\theta}{2\pi}.$$

Appendix B

Algorithm for the Visibility Criterion:

1. Get θ_{MAX} from ratio equation.
2. If θ_{MAX} is 2π, then stop, criterion is satisfied.
3. Compute $\Omega_{\oplus}$.
4. Compute $N_{\theta_{MAX}}$ and $N_{\Omega_{\oplus}}$.
5. Compute and evaluate x_j for an integer value letting j range from 1 to $N_{\Omega_{\oplus}}$.

References

[1] HAYES, E. W. "Computation of Average Revisit Time for Earth Observing Satellites," Paper No. 88-0574, AIAA Aerospace Sciences Meeting, Reno, Nevada, January 11–14, 1988.

[2] HAYES, E. W. "Computing Average Coverage Access Time to Point and Area Targets by Earth Observing Satellites," Paper No. 88–4275, AIAA/ASS Astrodynamics Conference, Minneapolis, Minnesota, August 15–17, 1988.

[3] LANGE, T. J., and HANSOM, J. M. "Orbital Constellations which Minimize Revisit Time," AAS Paper No. 83-402, AAS/AIAA Astrodynamics Conference, Lake Placid, New York, August 22–25, 1983.

[4] DRAIM, J. E. "Three- and Four-Satellite Continuous-Coverage Constellations," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 8, No. 6, November-December 1985, pp. 725–730.

[5] ALFANO, S., NEGRON, JR., D., and MOORE, J. L. "Determination of Satellite-Satellite and Satellite-Ground Station Visibility Periods by the Method of Parabolic Blending," AAS/AIAA Spaceflight Mechanics Meeting, Johnson Space Center, Houston, Texas February 11–13, 1991.

[6] ESCOBAL, P. R. *Methods of Orbit Determination*, Robert E. Krieger Publishing Company, Melbourne, Florida, 1965, p. 369.